GENERALIZATION OF CONVENTIONAL BCS MODEL TO ISO-SUPERCONDUCTIVITY MODEL OF HIGH-Tc SUPERCONDUCTIVITY IN THE CUPRATES AND PNICTIDES†

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Abstract

After years of successful application of the pseudo and model potential representation of electron-phonon interaction to conventional Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity, herein called the standard model, we have developed a generalization (herein called isostandard or iso-superconductivity model) that not only explains the differences between conventional and high-Tc superconductivity in the cuprates but also permits, in this paper, successful applications to the new high-Tc iron pnictides and to MgB2.

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1. INTRODUCTION

The discoveries by Berdnoz and Muller1 in 1986 at IBM Zurich of superconducting phase transition in a family of ceramic oxide materials and by Wu et al2 in 1987 in the 1-2-3 compound, (cuprates with structural formula, \((...)Cu_{n}O_{n-x}\)) at rather high critical temperatures (Tc) of 35K and 95K respectively, opened up the field of experimental and theoretical high-Tc superconductivity research which has remained very active to date. In addition to the 2001 discovery3 of high-Tc of order 35K in MgB2, the more recent discovery of superconductivity in the iron-based compounds4 - iron oxypnictides/single


layered LnOMPn (Ln = La, Ce, Pr, Nd, Sm, Gd, Tb, Dy, Ho and Y; M = Mn, Fe, Co and Ni: Pn = P and As)\(^5\); oxyfree-pnictides/single layered AMnPn (A = LnO = Li and Na; M = Mn, Fe, Co and Ni; Pn = P and As)\(^6\); oxyfree-pnictides/double layered ALM\(_2\)Pn\(_2\) (AL = Ba, Sr, Ca; M=Mn, Fe, Co and Ni; Pn = P and As)\(^7\,8\) and chalcogen/nonlayered MCn (M = Mn, Fe, Co and Ni; Cn = S, Se and Te)\(^9\,10\) has heightened interest in developing a suitable generalization of the Bardeen-Cooper-Schrieffer (BCS) model of superconductivity in simple metallic systems for understanding the coexistence\(^11\) of superconductivity and magnetic order in these solid compounds. The prospect of finding such a generalization is now brighter than ever for the following reason: just as high-T\(_C\) superconductivity in the cuprates is known to be a two-band phenomenon involving the copper 3d and oxygen 2p bands of the CuO\(_2\) planes, but was reduced to an effective single band pairing problem by Anderson\(^12\) in 1987 via his doped resonant valence bond (RVB) model and its generalization by Zhang and Rice\(^13\) in 1988 to the t–J model, so also is high- T\(_C\) superconductivity in the iron-based compounds known to involve multi-orbital effects of the Fe-3d with filling of approximately six electrons per Fe-site in the pnictides, but has been shown in the 2009 selfconsistent fluctuation exchange (FLEX) model by Zhang et al\(^14\) to be reducible to an orbital s(\(\uparrow\), \(\downarrow\)) coupling affair also known as the \(s_\pm\) state\(^15\).

Our proposed generalization in this paper is based on the observation by Animalu\(^16\) in 1991 and its elaboration\(^17\) in 1994 under the name iso-superconductivity that the Cooper pair of the standard BCS model may have a nonlocal nonhamiltonian structure CP = (\(e^-\uparrow\), \(e^-\downarrow\))\(_{HM}\) equivalent to the strong interaction (“hadronic” mechanics (HM)) structure of the neutral pion, as compressed positronium atom, \(\pi^0 = (e^+\uparrow\), \(e^-\downarrow\))\(_{HM}\), proposed by Santilli\(^18\) in 1978, i.e. an extended structure arising from the mutual overlapping/penetration of the wavefunctions of the constituents of the pair. The result is an effective generalization of the pseudo or model potential representation of electron-phonon interaction in the standard BCS model in such a way that whereas the Feynman graph for electron-electron scattering leading to net attraction, i.e., Cooper pairing in the BCS model is mediated by virtual phonon exchange (as shown in Fig. 1a), the situation in the iso-superconductivity model for a high-TC cuprate materials, (...Cu\(_m\)O\(_{n-x}\), is that a Cu\(^{2+}\) ion of effective valence, \(z = 2(n - x)/m\) (or Fe\(^{2+}\) for appropriate value of \(z\) in the iron pnictides) provides a "trigger" for the overlapping (i.e., "covalent" mixing) of electron wavefunctions to form a singlet pair, (\(e^-\downarrow\), \(e^-\uparrow\))\(_{HM}\) (see, Fig.1b). The main features of the transition from the BCS
model to the isosuperconductivity model and its prediction for Tc will be presented in Sec.2.

In models that propose to unite superconducting and antiferromagnetic phases in a larger symmetry group, SO(2N), an outstanding problem has been their inconclusive nature due to the inability to exactly diagonalize the model Hamiltonians. However, in 1992 C.N. Animalu19 proposed an alternative exact method for diagonalizing any second-quantized Hamiltonian model for arbitrary N-electron system based on $2^N \times 2^N$-matrix representation of the electron creation and annihilation operators suggested in 1961 by Thouless20. Such a representation (which we shall skip for simplicity) is of considerable interest because it provides a $2^N$-dimensional spinor representation of the group, SO(2N), corresponding to the groups, SO(8) for N = 4 and SO(10) for N = 5, the latter being realized for a lattice of 32 $\times$ 32 sites that has been employed by Zhang et al14 in their 2009 solution of the FLEX equation on imaginary frequency axis. These SO(2N) groups are among the groups that matter in string/superstring theories21. In Sec. 3, we shall recapitulate the experimental verification of the predictions of Tc by the isosuperconductivity theory in the cuprates and present, for the first time, the

**FIG. 1:** (a) Attractive electron-electron interaction mediated by virtual phonon exchange in the conventional BCS model; (b) attractive electron-electron pairing due to overlapping of electron wave functions around Cu$^{2+}$ or Fe$^{2+}$ ion ”trigger” in orbital $s(\uparrow,\downarrow)$–state envisaged in isosuperconductivity model.
corresponding verification for the pnictides, as well as for MgB$_2$, leading as a consequence, to a useful semi-empirical formula for designing superconductors with Periodic Table-based maps and material databases in the current search for room temperature superconductors. Conclusions will be drawn in Sec. 4.

2. THE ISOSUPERCONDUCTIVITY MODEL

As a prelude to the definition and characterization of the isosuperconductivity model, we begin in this section with a brief review of the conventional/non-conventional models of superconductivity.

2.1 Review of Conventional and Non-conventional Models

Because the conventional/non-conventional quantum mechanics models of superconductivity are based on second quantization formalism in both Bloch (k-space) and Wannier (r-space) representations, it should be recalled that one can formally transform the BCS model for superconductivity at low temperatures given in the Bloch representation by

\[ H_{BCS} = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^+ c_{k\sigma} - \sum_{k<k'} V_{kk'} c_{k\uparrow}^+ c_{-k\downarrow} c_{-k\downarrow} c_{k\uparrow}, \]  

(2.1a)

into the Wannier representation

\[ H_{BCS}^W = -\sum_{\{i,j\}\sigma} t_{ij} (c_{i\sigma}^+ c_{j\sigma} c_{j\sigma}^+ c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}, \]  

(2.1b)

This is achieved by making the following substitutions:

\[ c_{k\sigma}^+ = N^{-\frac{1}{2}} \sum_j e^{i\mathbf{k} \cdot \mathbf{R}_j} c_{j\sigma}^+; \quad c_{k\sigma} = N^{-\frac{1}{2}} \sum_j e^{-i\mathbf{k} \cdot \mathbf{R}_j} c_{j\sigma}, \]

\[ \sum_{k\sigma} \varepsilon_k n_{k\sigma} = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^+ c_{j\sigma}; \quad t_{ij} = \frac{1}{N} \sum_k \varepsilon_k e^{i\mathbf{k} \cdot (\mathbf{R}_j - \mathbf{R}_i)}. \]  

(2.2)

where $n_{ij} = c_{i\sigma}^+ c_{j\sigma}$ and by using the following approximations

\[ \varepsilon_{kk'} = -2t_{kk'} \rightarrow t_{ij} = t; \quad -V_{kk} / N \rightarrow U_{ijkl} = U, \]  

(2.3)
where \(-t_{ij} = \langle \phi_j \left( -\frac{\hbar^2}{2m} \nabla^2 \right) \phi_i \rangle\) is the hopping matrix while the electron-electron interaction energy is derived from the general expression

\[
\frac{1}{2} \sum_{ijkl} U_{ijkl} c_j^+ c_k^+ c_i c_i
\]

where

\[
U_{ijkl} = \int d\mathbf{r}_1 d\mathbf{r}_2 \phi_j^*(\mathbf{r}_1) \phi_i(\mathbf{r}_1) V_{ee}(|\mathbf{r}_1 - \mathbf{r}_2|) \phi_k^*(\mathbf{r}_2) \phi_i(\mathbf{r}_2)
\]

is the matrix element of the effective Coulomb interaction between Wannier states on different sites, \(\phi_i(\mathbf{r}) \equiv \phi(\mathbf{r} - \mathbf{R}_i)\). Finally, from the Hubbard-type Hamiltonian form in Eq.(2.1b), the \(t-J\) model for high-\(T_C\) close to half-filling is abstracted in the form

\[
H_{t-J} = -t \sum_{\langle ij \rangle, \sigma} \left[ \left( 1 - n_{i\sigma} \right) c_{i\sigma}^+ c_{j\sigma} \left( 1 - n_{j\sigma} \right) + h.c. \right] + J \sum_{\langle ij \rangle} \left( s_i \cdot s_j - \frac{1}{4} n_i n_j \right)
\]

(2.4)

### 2.2. Definition of the Isosuperconductivity Model

In its simplest non-relativistic form, the isostandard model of superconductivity\(^{16,17}\) is a generalization of the Lurie-Cremer\(^{12}\) quasiparticle wave equation,

\[
i \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = H \Psi(\mathbf{r}, t), \quad H \equiv \frac{1}{2m} \mathbf{p}^2 \tau_3 + \Delta \tau_1
\]

(2.5)

via the non-unitary ("isotopic lifting") transformation of the underlying "metric" \((g)\),

\[
g \equiv \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -T \end{pmatrix} \equiv \hat{g} \quad (2.6)
\]

which is characterized by a nonlocal integral (pseudopotential) opertar defined by

\[
T \psi^*_\downarrow(\mathbf{r}) = \int d^3r' \left[ \mathcal{D}(\mathbf{r} - \mathbf{r}') - \psi^*_\uparrow(\mathbf{r}) \psi^*_\uparrow(\mathbf{r}') \right] \psi^*_\downarrow(\mathbf{r}')
\]

(2.7)
where \( \psi^\dagger_\uparrow(r) \) and \( \psi_\downarrow(r) \) are the two spinor components of the quasi-particle wavefunction \( \Psi(r,0) \) in the Nambu representation, \( p^2/2m \) being the kinetic energy operator (measured from the Fermi level) and \( \Delta \) is the pair potential energy. It is apparent from Eq. (2.7) that when the overlap integrals or "orthogonalization term"

\[
Z^\dagger = \int d^3r' \psi^\dagger_\downarrow(r') \psi_\uparrow(r') \equiv \langle \psi^\dagger_\downarrow | \psi_\uparrow \rangle
\]

is zero, \( T \) reduces to unity and we recover the standard (BCS) model exactly.

Since we may rewrite \( T \) in the form

\[
T = 1 - \left| \psi^*_\downarrow \right| \left| \psi_\uparrow \right|
\]

so that \( T^2 = T \) if \( \left| \psi^*_\downarrow \right| \left| \psi_\uparrow \right| \neq 0 \), the physical effect of \( T \) is that the charge on the \( e^- \uparrow \) represented by the expectation value of \( T \), i.e.,

\[
\langle \psi^*_\downarrow | T | \psi_\uparrow \rangle = 1 - Z
\]

is "depleted" by an amount \( Z \) (called the "orthogonalization charge") whereas the charge on \( e^- \downarrow \) appears to vanish, i.e.,

\[
\langle \psi^*_\uparrow | T | \psi_\uparrow \rangle = 0
\]

In other words, \( e^- \uparrow \) behaves like a neutral spin-\( \frac{1}{2} \) quasiparticle (spinion) while \( e^- \downarrow \) behaves like a fractionally-charged quasiparticle ("anyon").

Consequently, in the solid state where the wavefunction \( \psi_\sigma(r,t) \) to which the nonlocal transformation in Eq.(2.6) is to be applied is related to the \( \phi_i(r) \) and \( c_{k\sigma}(t) \) of the second-quantized formulation by

\[
\psi_\sigma(r,t) = \sum c_{k\sigma}(t) \phi_i(r)
\]

the corresponding transformation of the corresponding creation and annihilation operators, \( c_{k\sigma}^+ \) and \( c_{k\sigma} \), into iso-creation and iso-annihilation operators, is defined by

\[
\hat{c}_{ik\sigma}^+= T_{ik\sigma} c_{ik\sigma}^+ \equiv (1-n_{ik\sigma}) \quad n_{ik\sigma} = c_{ik\sigma}^+ c_{ik\sigma}
\]

and similarly for \( \hat{c}_{ik\sigma} \).
\[-t \sum_{\langle ij \rangle, \sigma} \hat{c}_{i\sigma}^+ \hat{c}_{j\sigma} = -t \sum_{\langle ij \rangle, \sigma} (1 - n_{i\sigma}) c_{i\sigma}^+ c_{j\sigma} (1 - n_{j\sigma}). \tag{2.12}\]

as in Eq.(2.4) characterizing the $t - J$ model. It follows that the difference between the $t - J$ model and the isosuperconductivity model lies in the replacement of the U-term in Eq.(2.1b) by the J-term in the $t - J$ model (with the antiferromagnetic exchange constant $J = t^2 / U$ via second-order perturbation theory). Typically, $i(j) = d, p$ label electrons (bands) of Cu 3d and/or O 2p characters whose wavefunctions may overlap and/or bands hybridize; and $(i(j) = 1, 2, ..., N)$ in the nearest-neighbour electron transfer (hopping) integral.

By virtue of the transformation defined by Eq.(2.11), only single occupancy per spin site is permitted but double occupancy of an orbital site is not forbidden. Another feature of the second-quantized theory form of the iso-creation and iso-annihilation operators is that the waveoverlapping is associated with the coexistence of a non-zero antiferromagnetic spin wave state, $\langle c_{i\uparrow}^+ c_{i\downarrow} \rangle \neq 0$ and Cooper pair state $\langle c_{i\uparrow} c_{i\downarrow} \rangle \neq 0$ under Gor’kov’s factorization of the products of three fermion creation and annihilation operators involved in the transformation

\[T_{ij} c_{i\downarrow} = (1 - c_{i\downarrow}^+ c_{i\downarrow}) c_{i\downarrow}\]

\[\equiv c_{i\downarrow}^+ c_{i\downarrow} c_{i\downarrow} \rightarrow \langle c_{i\uparrow}^+ c_{i\downarrow} \rangle c_{i\downarrow} - \langle c_{i\uparrow} c_{i\downarrow}^+ \rangle c_{i\downarrow} + \langle c_{i\uparrow}^+ c_{i\downarrow} \rangle c_{i\downarrow} \tag{2.13}\]

In this (mean field) sense, one can derive from the isosuperconductivity model one of the primary objectives of the $t - J$ model which is to describe the coexistence of superconductivity and antiferromagnetism in high-$T_C$ materials as a function of band filling. The most important difference between the $t - J$ model and the isosuperconductivity model lies in the ability of the latter to predict $T_C$ from an exact solution of the model, to which we now turn.

### 2.3. Prediction of $T_C$

In conventional BCS model, the determination of the critical temperature for superconductivity involves solving an integral equation for the energy gap. But the beauty of the isosuperconductivity model is that instead of an integral equation, the desired result comes from the self-consistent solution of the conventional Schrodinger equation for one spin state, $\langle \psi_{\downarrow} \rangle$ say.
\( H\psi_{\downarrow} \equiv \left( \frac{p^2}{2m} + V_C \right)\psi_{\downarrow} = E_{\downarrow}\psi_{\downarrow}, \) \hspace{1cm} (2.15a)

in the Coulomb field \( V_C \) of the \( Cu^{2+} \) ion “trigger” in Fig. 1b, and an iso-Schrodinger equation

\( HT\psi_{\uparrow} \equiv \left( \frac{p^2}{2m} + V_H \right)\psi_{\uparrow} = E_{\uparrow}\psi_{\uparrow}, \) \hspace{1cm} (2.15b)

for the opposite spin state(\( \psi_{\uparrow} \)), where \( T \) is the non-local (psuedopotential) integral operator defined by Eq.(2.7). This has the effect of replacing the Coulomb potential, \( V_C \), by an effective Hulthen potential, \( V_H \) in Eq.(2.15b) for (e\( \downarrow \), e\( \uparrow \)) pairing in a singlet state:

\[ V_C \rightarrow V_C - \frac{E_{\downarrow}\langle \psi_{\downarrow} | \psi_{\uparrow} \rangle \psi_{\downarrow}}{\psi_{\uparrow}} = -V_0 \frac{1}{e^{\nu} - 1} \equiv V_H \] \hspace{1cm} (2.15c)

where \( V_0 \) is proportional to \( \langle \psi_{\downarrow} | \psi_{\uparrow} \rangle \). From the exact solution of Eq.(2.15b), Animalu\(^{16,17} \) derived the following formula for the critical temperature having the general form:

\[ T_c = \frac{\Theta_j}{\exp\left(\frac{1}{NV}\right) - 1} \] \hspace{1cm} (2.16a)

where \( NV \) represents the dimensionless coupling constant while

\[ \Theta_j = \frac{\hbar \omega_p}{k_B \sqrt{d \varepsilon(q_D)}} \] \hspace{1cm} (2.16b)

is the “jellium” temperature, \( d = 1, 2, 3 \) being the effective dimensionality of the system, and \( \varepsilon(q_D) \) the Hatree dielectric function evaluated at the Debye wavenumber \( q_D \). We observe that in the weak coupling limit \( NV < 1 \), we may express the result in the BCS form:

\[ T_c = \Theta_j \exp(-1/NV) \] \hspace{1cm} (2.17a)

But in the strong coupling limit, i.e. if \( NV > 1 \), we may expand the exponential in the denominator of Eq.(2.16a) to first order in \( 1/NV \) to get

\[ T_c = \Theta_j NV \] \hspace{1cm} (2.17b)

Our interest is to show how accurately these results agree with experimental data to which we now turn.
3. EXPERIMENTAL VERIFICATION

3.1. The Cuprates

An explicit form of Eq.(2.17a) used in Ref. 16 for the verification with experimental data in the cuprates with structural formula \( \text{ (...)Cu}_m\text{O}_{n-x} \) is:

\[
T_c = \Theta_J \exp\left(\frac{-13.6}{z}\right) = \left(\frac{367.3z}{\sqrt{d_{\varepsilon}}}\right) \exp\left(\frac{-13.6}{z}\right) \left(0 \text{~K}\right) \tag{3.1}
\]

where the effective valence \( z \) of the \( Cu^{z+} \) ion is given by \( z = \frac{2(n-x)}{m} \).

Table 1 Dependence of \( T_c \) on the effective valence \( z \) of \( Cu^{z+} \) in the cuprates and \( Fe^{z+} \) in the iron pnictides.

<table>
<thead>
<tr>
<th>( z )</th>
<th>( T_c(Cu^{z+}) )</th>
<th>( T_c(Fe^{z+}) )</th>
<th>( z )</th>
<th>( T_c(Cu^{z+}) )</th>
<th>( T_c(Fe^{z+}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0005</td>
<td>0.0006</td>
<td>4.1</td>
<td>54.6032</td>
<td>69.4247</td>
</tr>
<tr>
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<td>60.5318</td>
<td>76.9625</td>
</tr>
<tr>
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<td>1.0403</td>
<td>4.3</td>
<td>66.8201</td>
<td>84.9577</td>
</tr>
<tr>
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<td>3.9847</td>
<td>5.0663</td>
<td>4.4</td>
<td>73.4698</td>
<td>93.4125</td>
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<td>15.0535</td>
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</tr>
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<td>4.5</td>
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</tr>
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<td>38.4549</td>
<td>4.5</td>
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</tr>
<tr>
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<td>4.5</td>
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<td>102.328</td>
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<td>62.2413</td>
<td>4.5</td>
<td>80.4820</td>
<td>102.328</td>
</tr>
</tbody>
</table>
FIG. 2: Predicted dependencies of the Jellium temperature $\Theta_j$ and the superconducting transition temperature $T_c$ on the effective valence $z = 2(n - x)/m$ of $Cu^{\text{+}}$ ions in the family of compounds, $\ldots Cu_m O_{n-x}$ are compared with experimental data (*) as discussed in the ref.[17]. The experimental Debye temperatures of pure copper $\Theta_D (Cu)$ and pure vanadium $\Theta_D (V)$ are indicated.
As a further confirmation of the formula in Eq.(2.17a), we present in Tables 2-4 and Fig. 3 another set of experimental data on the high-Tc doped compounds\textsuperscript{23-25}, $\text{YBa}_2\text{Cu}_{3-x}\text{Mn}_x\text{O}_y$ and $\text{GdBa}_2(\text{Cu}_{1-x}\text{M}_x)_3\text{O}_{7-\delta}$, (M = Ni and Zn), in which the effects of the substitution of Cu by transition- and non-transition-metal ions are represented by the modification of the effective valance ($z$) on $\text{Cu}^{\text{i}+}$ indicated in tables 3-4. Again reasonable agreement between theory and experiment is obtained for $z$ lying in the range, $4.61 \geq z \geq 4.21$.

### Table 2. $\text{YBa}_2\text{Cu}_{3-x}\text{Mn}_x\text{O}_y$ (After Ref. 23)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$T_c$ (Theory)</th>
<th>$T_c$ (Expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>6.92</td>
<td>4.613</td>
<td>88.9</td>
<td>91</td>
</tr>
<tr>
<td>0.03</td>
<td>6.88</td>
<td>4.541</td>
<td>83.5</td>
<td>86.6</td>
</tr>
<tr>
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<td>4.447</td>
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<tr>
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<td>6.91</td>
<td>4.387</td>
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</tr>
<tr>
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<td>6.92</td>
<td>4.312</td>
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<td>72.0</td>
</tr>
<tr>
<td>0.30</td>
<td>6.95</td>
<td>4.212</td>
<td>61.3</td>
<td>67.0</td>
</tr>
</tbody>
</table>

Note: $T_c$(theory) is given by Eq.(3.1) where the effect of replacing $\text{Cu}_3$ by $\text{Cu}_{3-x}\text{Mn}_x$, is obtained by replacing 3 by $(3 - x) + 2x = 3 + x$ which lowers the effective valence ($z$) on $\text{Cu}^{\text{i}+}$ ions to $z = \frac{2y}{(3 + x)}$. 


FIG. 3: Predicted dependence of the superconducting transition temperature $T_c$ on the effective valence ($z$) of C$_{z+}$ ions (continuous curve) given by $T_c = 367.32 \exp(-13.6/z)$ in the doped 1:2:3 cuprates are compared with experimental data as discussed in Tables 1, 2, 3 and the text.
Table 3: GdBa$_2$(Cu$_{1-x}$Ni$_x$)$_3$O$_{7-\delta}$ (after ref. 24)

<table>
<thead>
<tr>
<th>x</th>
<th>y = 7 - $\delta$</th>
<th>z</th>
<th>$T_c$(Theory)</th>
<th>$T_c$(expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>6.96</td>
<td>4.640</td>
<td>91.0</td>
<td>91</td>
</tr>
<tr>
<td>0.025</td>
<td>6.96</td>
<td>4.527</td>
<td>82.5</td>
<td>79</td>
</tr>
<tr>
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<td>4.419</td>
<td>74.8</td>
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<tr>
<td>0.075</td>
<td>6.96</td>
<td>4.316</td>
<td>67.9</td>
<td>65.0</td>
</tr>
</tbody>
</table>

Note: $T_c$ (theory is given by Eq.(3.1) and $z = \frac{2y}{3(1+x)}$ as discussed in Table 2.

Table 4: GdBa$_2$(Cu$_{1-x}$Mn$_x$)$_3$O$_{7-\delta}$ (after ref.25)

<table>
<thead>
<tr>
<th>x</th>
<th>y = 7 - $\delta$</th>
<th>z</th>
<th>$T_c$(Theory)</th>
<th>$T_c$(expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>6.96</td>
<td>4.640</td>
<td>91.0</td>
<td>91</td>
</tr>
<tr>
<td>0.025</td>
<td>6.96</td>
<td>4.309</td>
<td>67.4</td>
<td>54</td>
</tr>
<tr>
<td>0.050</td>
<td>6.96</td>
<td>4.009</td>
<td>49.0</td>
<td>37</td>
</tr>
<tr>
<td>0.075</td>
<td>6.96</td>
<td>4.737</td>
<td>36.1</td>
<td>35</td>
</tr>
</tbody>
</table>

Note: $T_c$ (theory is given by Eq.(3.1) and $z = \frac{2y}{3[1+3(x+x^2)]}$ includes an extra $x^2$ term for the non-transition metal Zn ion substitution in order to give a reasonable phenomenological fit to the data.

3.2 The IronPnictides

In order to compare with experimental data in the iron pnictides, we now turn to a realization of the formula in Eq.(2.17b) in a similar form:

$$T_c = 467.0z\exp\left(-\frac{13.6}{z}\right)^0 K;$$
where $467.0 = \Theta_D$ is the experimental Debye temperature of iron (see Table 1). It is also plotted alongside the result for the cuprates in Fig. 4. There is good agreement with the experimental data in the pnictide, $\text{Ba}_0.6\text{K}_{0.4}\text{Fe}_2\text{As}_2$ from neutron scattering\(^7\) (see, Fig. 5).

**FIG. 4:** (colour online) Predicted dependence of the transition temperature $T_c$ on the effective valence $z$ for the cuprates in Eq.(3.1) and the pnictides in Eq.(3.2)
3.3 MgB$_2$

For MgB$_2$, the corresponding prediction is

$$T_C = 406.0 z \exp\left(- \frac{13.6}{z}\right) K$$ \hspace{1cm} (3.3)

where 406.0 = $\Theta_D$ is the experimental Debye temperature of Mg. The results are tabulated in Table 6 which shows that the observed $T_C$ of (39 K) corresponds to a value of $z$ close to 3.75.
Table 6: Dependence of $T_c$ on the effective valence of $\text{Mg}^{z+}$ in $\text{MgB}_2$

<table>
<thead>
<tr>
<th>$z$</th>
<th>$T_c$</th>
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<tr>
<td>1.00</td>
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<tr>
<td>1.50</td>
<td>0.0703</td>
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</table>

4. DISCUSSION AND CONCLUSION

By the time the iso-superconductivity (iso-standard) model was proposed by Animalu$^{16,17}$, the BCS model had lost predictive power for the available experimental data in the cuprates. Subsequently, when the highest $T_c$ of 165K so far in the cuprates was reported in 1994$^{25}$, the result was also in agreement with the iso-standard model prediction$^{26}$. The successful application of the iso-standard model in this paper to the prediction of the recent data on the iron pnictides lends further credence to the iso-standard model even though the effective valence ($z$) on Fe$z^+$ has been treated as a phenomenological parameter. We are therefore led to the conclusion that more serious studies of the foundation of iso-superconductivity in the analogy between Santilli’s model of the neutral pion as a compressed postronium atom and the iso-standard model of the Cooper pair should be undertaken: this is the subject-matter of the isotopic branch of
“hadronic” mechanics with far-reaching implications for quantum physics in the 21st C.

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